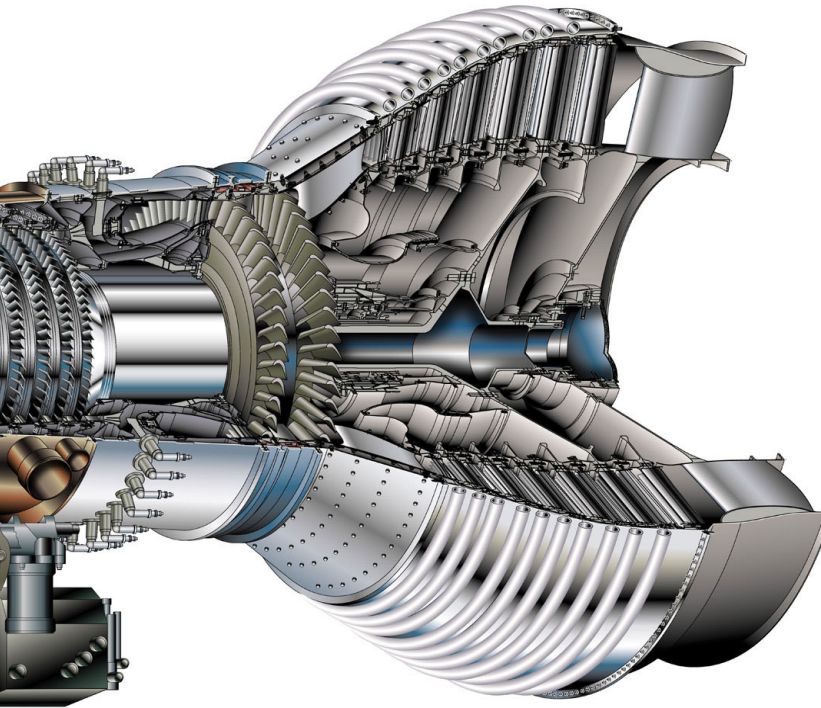


7. Engine Component Operation: Gas Turbine

Purpose

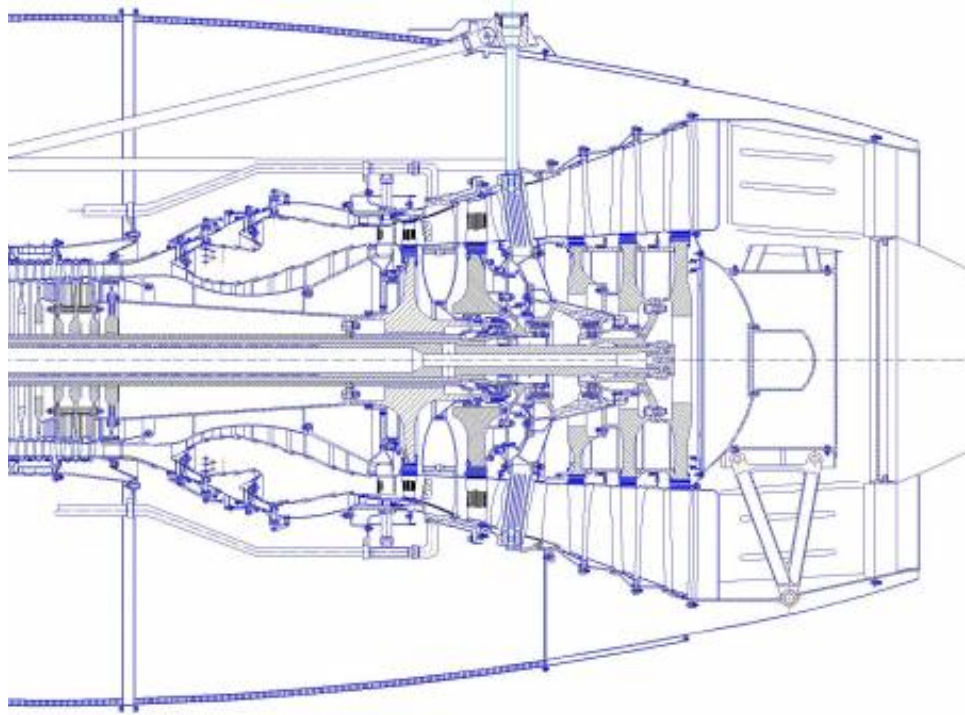


A turbine extracts power from the gas stream to drive compressors or/and another loads (propeller, rotors, electrical generator).

Classification

Turbines can be classified:

- by the number of cascades:
 - single-spool
 - multi-spool
- by number of stages
 - single-stage
 - multi-stage
- by the type of stage:
 - axial
 - radial



Operational parameters

π_t^* - turbine pressure ratio.

π_t^* is the ratio of total pressure values at inlet and exit of the turbine

$$\pi_t^* = \frac{p_4^*}{p_5^*}.$$

$n, \text{ min}^{-1}$ - rotational speed (speed of revolution).

$n_{\text{corr.4}}, \text{ min}^{-1}$ - corrected rotational speed of the turbine — rotational speed of the turbine rotor corrected to the standard conditions at the turbine inlet.

Rotational speed and corrected rotational speed are related by expression

$$n_{\text{corr.4}} = n \sqrt{\frac{288}{T_4^*}}.$$

Efficiency parameters

η_t^* - turbine isentropic efficiency.

Turbine isentropic efficiency (turbine efficiency) is a ratio of actual work done of the turbine L_t to its value at the isentropic (ideal) expansion process L_{ts} :

$$\eta_t^* = \frac{L_t}{L_{ts}} .$$

Actual and ideal processes must correspond to equal values of pressure ratio π_t^* .

Efficiency of axial turbines depends on the type of stages, aerodynamic quality, type of cooling system and dimensions

.

Stage type	η_t^*	$L_{t.stage} , \frac{kJ}{kg}$
moderate-loaded	0,90 ... 0,93	100 ... 200
high-loaded	0,87 ... 0,90	450 ... 550

The upper values of η_t^* in the recommended ranges apply to multi-stage turbines with a high level of aerodynamic quality, the lower – to single-stage turbines.

Small-scale turbines ($G_4 < 5 \dots 10 \frac{\text{kg}}{\text{s}}$) have reduced efficiency ($\Delta \eta_t^* = -(1 \dots 5)\%$).

Efficiency of high-temperature cooled turbines depends on the amount of cooling air. It decreases with increasing $g_{\Sigma \text{cooling}}$. It can be assumed that 1 % of cooling air ($g_{\text{cooling}} = 0,01$) leads to 1 % efficiency decreasing of the single-stage turbine, while efficiency of the two-stage (z-stage) turbine with cooled first stage is reduced by 1/2 % (1/z %).

μ_t - throughput capacity factor.

μ_t is the ratio of actual mass flow rate of the working fluid through the first stationary blade row (nozzle) to the ideal mass flow rate under the same conditions:

$$\mu_t = \frac{G_{th}}{G_{th,ideal}}.$$

a_t - relative throughput capacity.

$$a_t = \mu_t q(\lambda_{th}) = \frac{G_4 \cdot \sqrt{T_4^*}}{m \cdot p_4^* \cdot F_{th}}.$$

A_t - throughput capacity.

A_t allows to estimate amount of mass flow rate passing through the turbine under specified conditions at the inlet

$$A_t = \frac{G_4 \cdot \sqrt{R \cdot T_4^*}}{p_4^*}.$$

$\mu_t q(\lambda_{th}) F_{th}$ - throughput capacity.

$$\mu_t q(\lambda_{th}) F_{th} = \frac{G_4 \cdot \sqrt{T_4^*}}{m \cdot p_4^*}.$$

$$\mu_t q(\lambda_{th}) F_{th} \sim A_t.$$

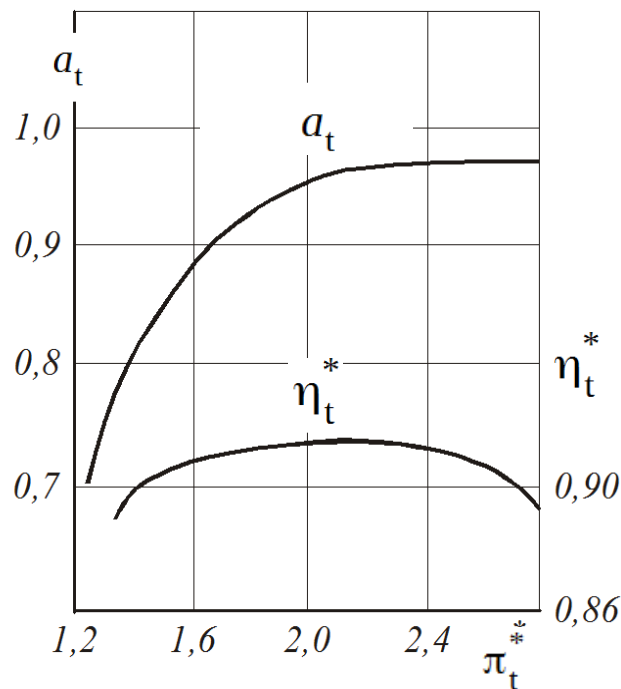
Turbine maps are usually represented as dependencies of efficiency parameters on two operational parameters. For example, they can be represented as dependencies of turbine efficiency η_t^* and relative throughput capacity a_t on turbine pressure ratio π_t^* and corrected rotational speed $n_{\text{corr},4}$.

However, efficiency dependencies on $n_{\text{corr},4}$ are insignificant at basic operating modes of multi-shaft engines.

Therefore, turbine maps with sufficient accuracy can be expressed as functions of one variable:

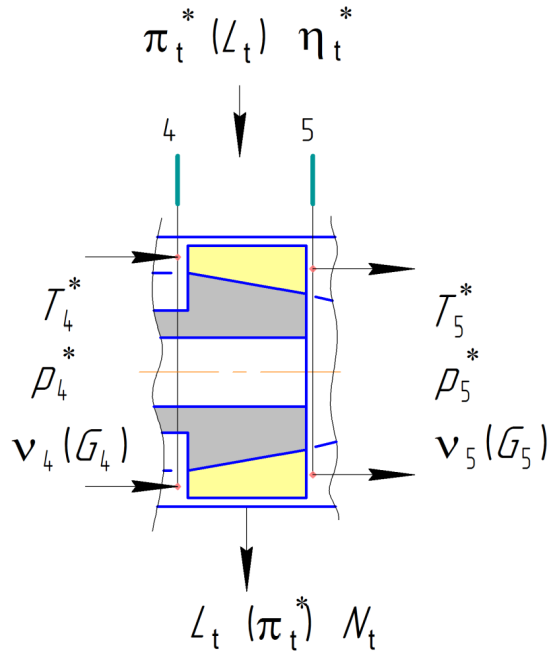
$$\eta_t^* = f(\pi_t^*),$$

$$a_t = f(\pi_t^*).$$

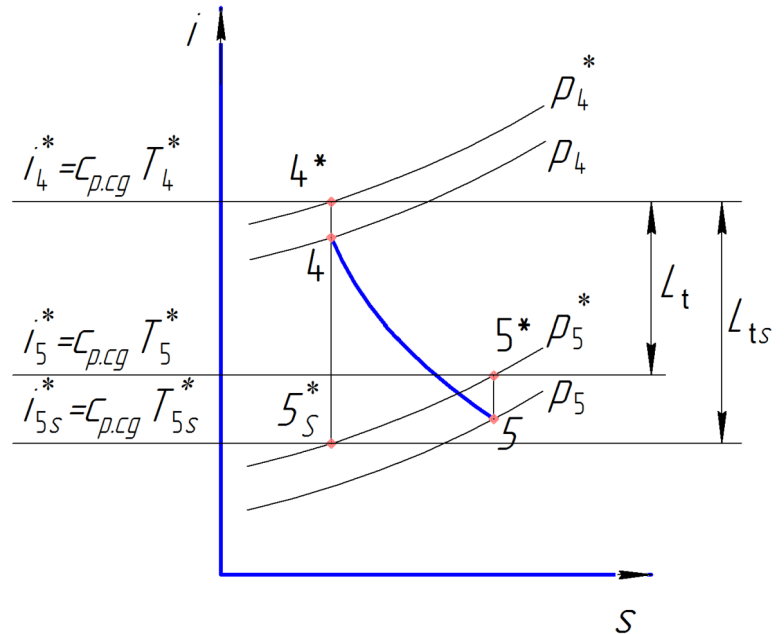


Turbine operating

Calculation Scheme



i-s diagram



Power Equilibrium — at steady state operation of the engine the power produced by the turbine with deduction of mechanical losses equals to the total power of all consumers connected to the turbine shaft.

Equation of Power Equilibrium:

$$N_{\text{cons.}\Sigma} = N_t \cdot \eta_m .$$

η_m - mechanical loss factor.

$$\eta_m = \frac{N_{\text{cons.}\Sigma}}{N_t} = 1 - \frac{N_{\text{losses}}}{N_t} .$$

For the high-pressure spool: $\eta_{m.\text{hp}} = 0,985 \dots 0,995$.

For other spools: $\eta_m = 1$.

Small-scale engines have increased mechanical losses ($\Delta \eta_m^* = -(1 \dots 5) \%$).

Compressor $\leftarrow$ Turbine:

$$N_c = N_t \cdot \eta_m ;$$

$$G_2 \cdot L_k = G_4 \cdot L_t \cdot \eta_m ;$$

$$L_t = \frac{L_k}{v_i \cdot \eta_m} , \text{ where } v_i = v_4 .$$

Fan $\leftarrow$ Turbine (low-pressure spool of turbofan engine):

$$N_{\text{fan}} = N_{\text{lpt}} \cdot \eta_{m.\text{lp}} ;$$

$$G_{21} \cdot L_{\text{lpc}} + G_{12} \cdot L_{\text{f II}} = G_{46} \cdot L_{\text{lpt}} \cdot \eta_{m.\text{lp}} ;$$

$$L_{\text{lpt}} = \frac{L_{\text{lpc}} + m \cdot L_{\text{f II}}}{v_i \cdot \eta_{m.\text{lp}}} , \text{ where } v_i = v_{46} .$$

Prop + Compressor $\leftarrow$ Turbine (turboprop engine):

$$N_e + N_c = N_t \cdot \eta_m ;$$

$$N_e + G_2 \cdot L_c = G_4 \cdot L_t \cdot \eta_m ;$$

$$N_{e.spec} = \frac{N_e}{G_2} = L_t \cdot v_i \cdot \eta_m - L_c, \text{ where } v_i = v_4 .$$

Load $\leftarrow$ Free Power Turbine (turboshaft engine):

$$N_e = N_{fpt} \cdot \eta_m ;$$

$$N_e = G_{46} \cdot L_{fpt} \cdot \eta_{m.fp} ;$$

$$N_{e.spec} = \frac{N_e}{G_2} = L_{fpt} \cdot v_i \cdot \eta_{m.fp}, \text{ where } v_i = v_{46} .$$

Pressure Equilibrium — at steady state operation of the engine the total compression pressure ratio equals to the total expansion pressure ratio.

Equation of Pressure Equilibrium:

$$\pi_{\text{compression}} = \pi_{\text{expansion}} \cdot$$

Primary flowpath of two-spool turbofan (turbojet) engine:

$$\pi_V \cdot \sigma_{\text{intake}} \cdot \pi_{\text{lpc}}^* \cdot \pi_{\text{hpc}}^* \cdot \sigma_b = \pi_{\text{hpt}}^* \cdot \pi_{\text{lpt}}^* \cdot \pi_{\text{n.c}} \cdot$$

Secondary flowpath of turbofan engine:

$$\pi_V \cdot \sigma_{\text{intake}} \cdot \pi_{\text{fII}}^* \cdot \sigma_{\text{II}} = \pi_{\text{nII.c}} \cdot$$

Single-spool turboshaft engine:

$$\pi_V \cdot \sigma_{\text{intake}} \cdot \pi_c^* \cdot \sigma_{\text{cc}} = \pi_{\text{hpt}}^* \cdot \pi_{\text{fpt}}^* \cdot \pi_{\text{n.c}}$$

Ideal expansion process:

$$\Delta i_{ts}^* = L_{ts} = i_4^* - i_{5s}^* = c_{p.cg} (T_4^* - T_{5s}^*);$$

$$L_{ts} = c_{p.cg} \cdot T_4^* \left(1 - \frac{T_{5s}^*}{T_4^*} \right) = c_{p.cg} \cdot T_4^* \left(1 - \left(\frac{p_5^*}{p_4^*} \right)^{\frac{k-1}{k}} \right);$$

$$L_{ts} = c_{p.cg} \cdot T_4^* \left(1 - \frac{1}{\pi_t^*^{\frac{k-1}{k}}} \right).$$

Actual expansion process:

$$\Delta i_t^* = L_t = i_4^* - i_5^* = c_{p.cg} (T_4^* - T_5^*);$$

$$L_t = L_{ts} \cdot \eta_t^*.$$

Uncooled Turbine Performance Calculation

Turbine pressure ratio:

- by Equation of Pressure Equilibrium;
- depending on L_t calculated by Equation of Power Equilibrium:

$$\pi_t^* = \left(1 - \frac{L_t}{c_{p.cg} \cdot T_4^* \cdot \eta_t}\right)^{-\frac{k}{k-1}}.$$

Turbine work:

- by Equation of Power Equilibrium with $v_i = v_4$.;
- depending on π_t^* calculated by Equation of Pressure Equilibrium:

$$L_t = c_{p.cg} \cdot T_4^* \cdot \left(1 - \frac{1}{\pi_t^{*\frac{k}{k-1}}}\right) \cdot \eta_t^*, \frac{\text{kJ}}{\text{kg}}.$$

Total temperature at the outlet:

$$T_5^* = T_4^* - \frac{L_t}{c_{p.cg}}, \text{ K} .$$

Total pressure at the outlet:

$$p_5^* = \frac{p_4^*}{\pi_t^*}, \text{ kPa} .$$

Relative mass flow rate at the outlet:

$$v_5 = v_4 .$$

Mass flow rate at the outlet:

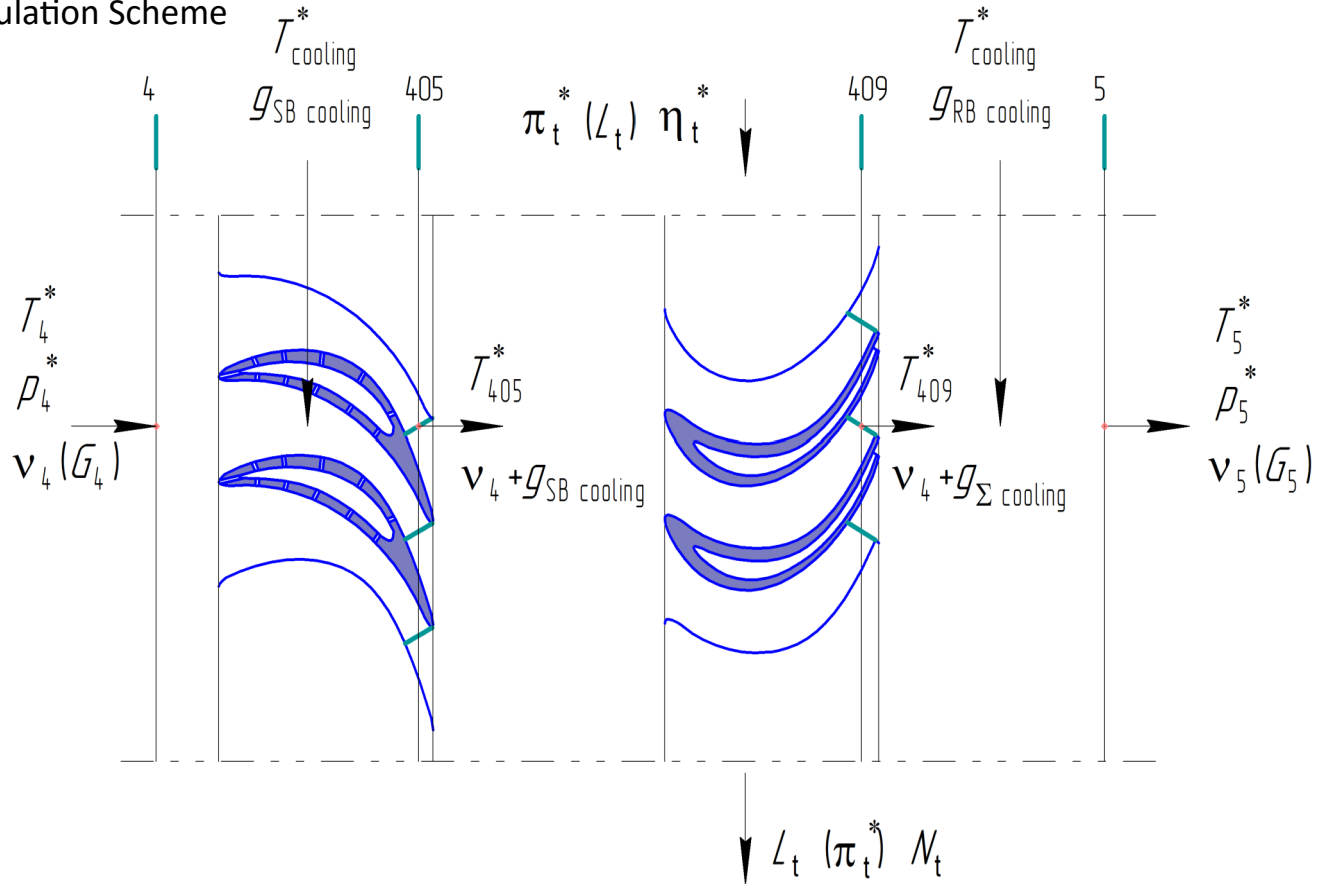
$$G_5 = G_4, \frac{\text{kg}}{\text{s}} .$$

Turbine power:

$$N_t = G_4 \cdot L_t, \text{ kW} .$$

Cooled Turbine Performance Calculation

Calculation Scheme



Total temperature at the throat of first stationary blade row:

$$T_{405}^* = \frac{c_{p.cg} \cdot T_4^* \cdot v_4 + c_{p.a} \cdot T_{cooling}^* \cdot g_{SB\ cooling}}{c_{p.cg} \cdot (v_4 + g_{SB\ cooling})}.$$

Turbine pressure ratio:

- by Equation of Pressure Equilibrium;
- depending on L_t calculated by Equation of Power Equilibrium:

$$\pi_t^* = \left(1 - \frac{L_t}{c_{p.cg} \cdot T_{405}^* \cdot \eta_t^*}\right)^{-\frac{k}{k-1}}.$$

Turbine work:

- by Equation of Power Equilibrium with $v_i = v_4 + g_{SB\ cooling}$;
- depending on π_t^* calculated by Equation of Pressure Equilibrium:

$$L_t = c_{p.cg} \cdot T_{405}^* \cdot \left(1 - \frac{1}{\pi_t^* \frac{k-1}{k}}\right) \cdot \eta_t^*, \frac{\text{kJ}}{\text{kg}}.$$

Total temperature at the throat of last rotational blade row:

$$T_{409}^* = T_{405}^* - \frac{L_t}{c_{p.cg}}, \text{ K}.$$

Total temperature at the outlet:

$$T_5^* = \frac{c_{p.cg} \cdot T_{409}^* \cdot (v_4 + g_{SB \text{ cooling}}) + c_{p.a} \cdot T_{cooling}^* \cdot g_{RB \text{ cooling}}}{c_{p.cg} \cdot (v_4 + g_{SB \text{ cooling}} + g_{RB \text{ cooling}})}, \text{ K}.$$

Total pressure at the outlet:

$$p_5^* = \frac{p_4^*}{\pi_t^*}, \text{ kPa}.$$

Relative mass flow rate at the outlet:

$$v_5 = v_4 + g_{\text{SB cooling}} + g_{\text{RB cooling}} .$$

Mass flow rate at the outlet:

$$G_5 = G_4 \frac{v_4 + g_{\text{SB cooling}} + g_{\text{RB cooling}}}{v_4} , \frac{\text{kg}}{\text{s}} .$$

Turbine power:

$$N_t = G_4 \frac{v_4 + g_{\text{SB cooling}}}{v_4} \cdot L_t , \text{ kW} .$$